Lecture 19

Tuesday, October 25, 2016 8:55 AM

4.7 Optimization Problems

First Derivative Test for extreme values

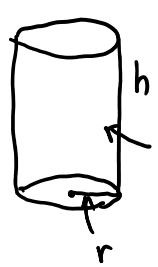
Spse c is a C.N. of continuous fn f defined on an interval :

a) If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the abs max of f.

b) If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the abs min of f.

Ex A cylindrical can is to be made to hold 1 L oil. Find the dimension that will minimize cost of manufacturing?

Soln

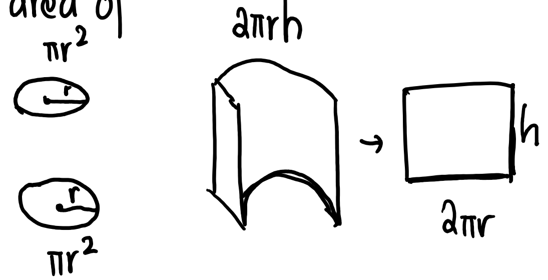


$r \equiv$ radius

$h \equiv$ height

GOAL Minimize the surface area of the cylinder.

$$\boxed{A = 2\pi r^2 + 2\pi r h}$$



Volume $= 1\,L = 1000\ cm^3$

$$V = \pi r^2 h = 1000 \Rightarrow h = \frac{1000}{\pi r^2}$$

$$A = 2\pi r^2 + 2\pi r \cdot \frac{1000}{\pi r^2}$$

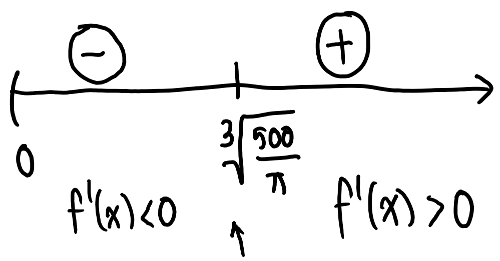
$$A(r) = 2\pi r^2 + \frac{2000}{r}, \quad r > 0$$

Find C.N

$$A'(r) = 4\pi r - \frac{2000}{r^2} = \frac{4(\pi r^3 - 500)}{r^2}$$

- $\pi r^3 - 500 = 0 \Rightarrow \pi r^3 = 500$

$$\Rightarrow r = \sqrt[3]{\frac{500}{\pi}}$$



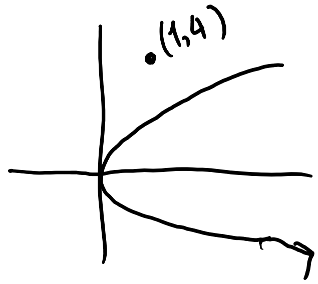
Abs min @ $r = \sqrt[3]{\frac{500}{\pi}}$

$$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2} \overset{\text{DIY}}{=} 2 \cdot \sqrt[3]{\frac{500}{\pi}} \;(= 2r)$$

Alternate Soln (Using Implicit Diffn)

Textbook.

Ex Find the point on parabola $y^2 = 2x$,
that is closest to the point $(1,4)$.



(x,y), $(1,4)$

$$d = \sqrt{(x-1)^2 + (y-4)^2}$$

Since (x,y) lies on the parabola, $x = \frac{y^2}{2}$

Minimize

$$d = \sqrt{\left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2}$$

CIY Since distance is always positive, minimizing
d is the same as minimizing d^2.

$$d^2 = \left(\frac{1}{2}y^2 - 1\right)^2 + (y-4)^2 = f(y)$$

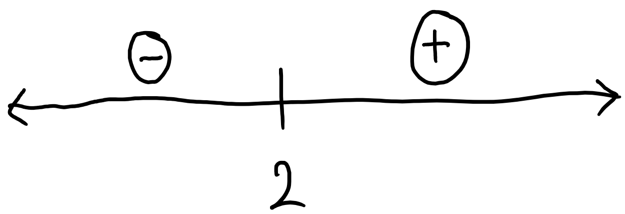
$$f'(y) = 2\left(\frac{1}{2}y^2 - 1\right) \cdot y + 2(y-4) \cdot 1$$

$$= (y^2 - 2) \cdot y + 2y - 8$$

$$= y^3 - 2y + 2y - 8$$

$$= y^3 - 8$$

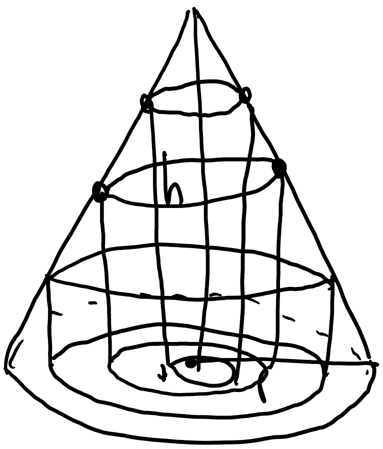
$$f'(y) = 0 \Rightarrow y^3 - 8 = 0 \Rightarrow y = \sqrt[3]{8} = 2$$

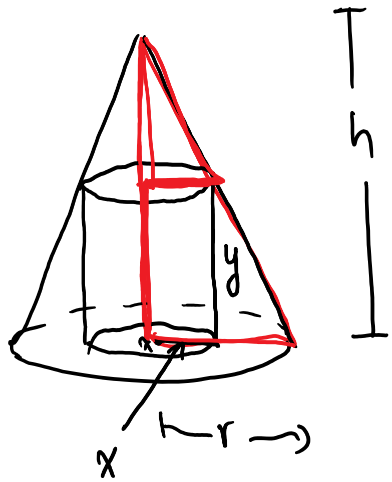


So abs min occurs when $y = 2$. $(2, 2)$

$$x = \frac{y^2}{2} = \frac{2^2}{2} = 2.$$

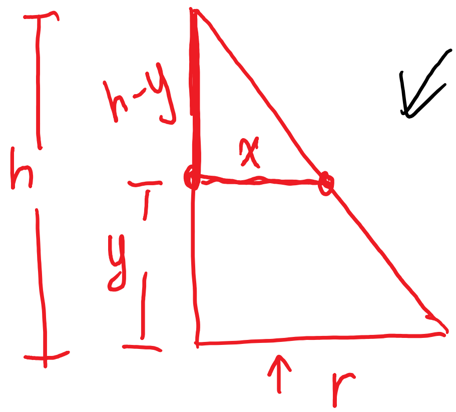
Ex A right circular cylinder is inscribed in a cone w/ ht h and radius r. Find the largest possible volume of such cylinder.

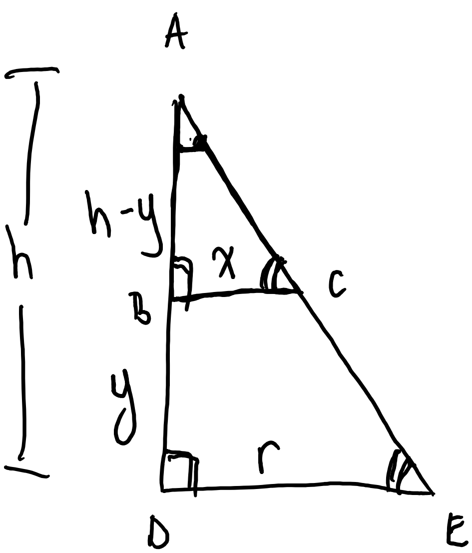




Maximize

$$V = \pi x^2 y$$





$\underline{\Delta ABC}$, $\underline{\Delta ADE}$ Similar triangles.

$$\frac{x}{r} = \frac{h-y}{h}$$

$$\Rightarrow hx = hr - ry$$

$$\Rightarrow ry = hr - hx$$

$$\Rightarrow y = \underline{hr - hx}$$

r

$= h - \frac{hx}{r}$

$V = \pi x^2 y$

$V(x) = \pi x^2 \left(h - \frac{hx}{r} \right)$ DIY